Kinaesthetic and visual interactive aids for mathematical researchers

Lunch Teaching Seminar, Uppsala University

Raazesh Sainudiin & Erik Gustavsson

October 14, 2021, Uppsala, SE

Abstract

I would like to show some concrete examples to aid students and/or teachers, understood more generally as mathematical researchers, to kinaesthetically and visually communicate mathematical concepts in interactions.

Such examples include:

- Galton's Quincunx and its three-dimensional extension known as the Septcunx (that can be 3D printed),
- A measurable double pendulum,
- SageMath to illustrate instances of Probability Theorems and Statistical Models in the Introduction to Data Science Course that Benny Avelin and I are currently teaching.

References:

https://escholarship.org/uc/item/8b15m415

https://www.isibang.ac.in/~statmath/eprints/2010/11.pdf

https://uppsala.instructure.com/courses/44135/pages/jupyter-notebooks?module_item_id=301399

Outline

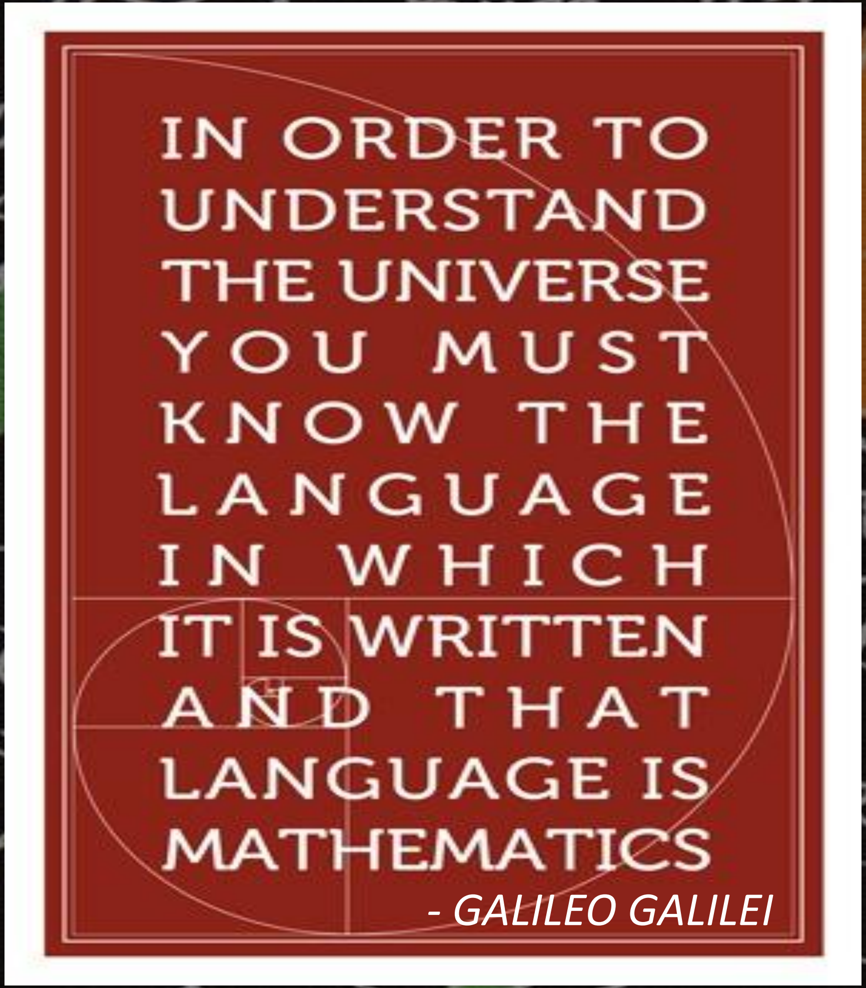
- Part 0: Overview of the “Why?”
- Part I: Kinaesthetic & Visual Cognitive Aids in Probability & Statistics
- Part II: Measurable Experiments & Mathematical Art
- Part III: SageMath for Theorems & Live Data in *Introduction to Data Science*

Part 0 - A high-level Overview of the “why?”

Why use Kinaesthetic and visual interactive aids for mathematical communications in lectures, tutorials/problem-sessions/recitations?

See More, Feel More, Do More:

Augmenting Communications in Mathematics Lectures and Tutorials with Visual, Tactile and Kinaesthetic Learning Aids


IN ORDER TO UNDERSTAND THE UNIVERSE YOU MUST KNOW THE LANGUAGE IN WHICH IT IS WRITTEN AND THAT LANGUAGE IS MATHEMATICS
- GALILEO GALILEI

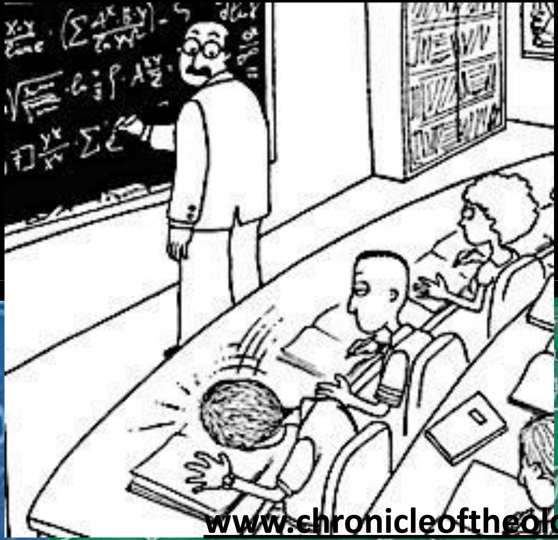
GOAL IN THE CLASSROOM

We need our students to

UNDERSTAND CONCEPTS CLEARLY,

Not just follow formulaic recipes

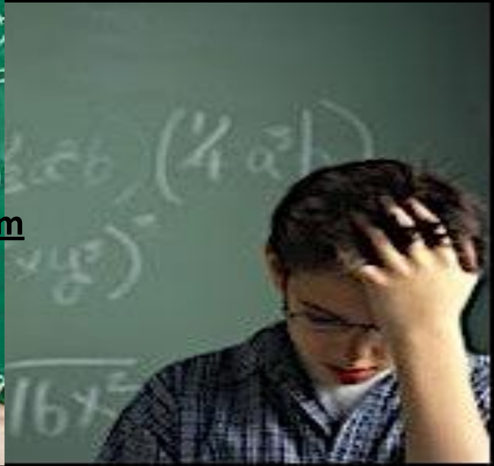
REALITY IN THE CLASSROOM



www.chronicleoftheoldwest.com

www.chemistryland.com

www.acclaimimages.com



www.bobgriggs.com

THE PROBLEM

- We have a limited time

  to cover a comprehensive syllabus

  to a large number of students
- Mathematics is a difficult language!

THE PROBLEM

WE MAY NEED TO COMMUNICATE BEYOND TRADITIONAL READ/WRITE METHODS

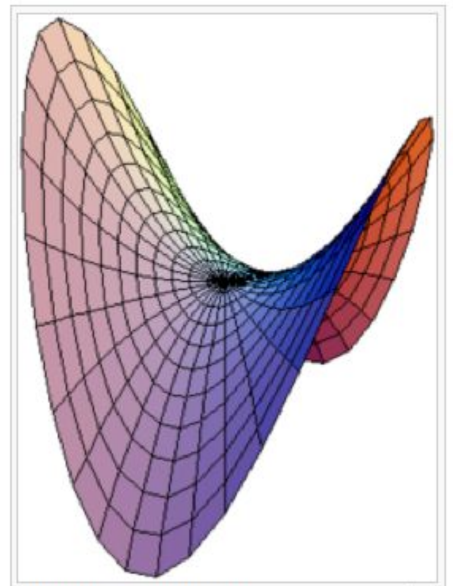
for the post-YouTube gen. bY USING MORE VISUAL, TACTILE & KINAESTHETIC STRATEGIES

Example: Hyperbolic Paraboloid

Traditional Read/Write: Formula

$$\frac{z}{c} = \frac{y^2}{b^2} - \frac{x^2}{a^2}.$$

Visual: Image



Hyperbolic paraboloid

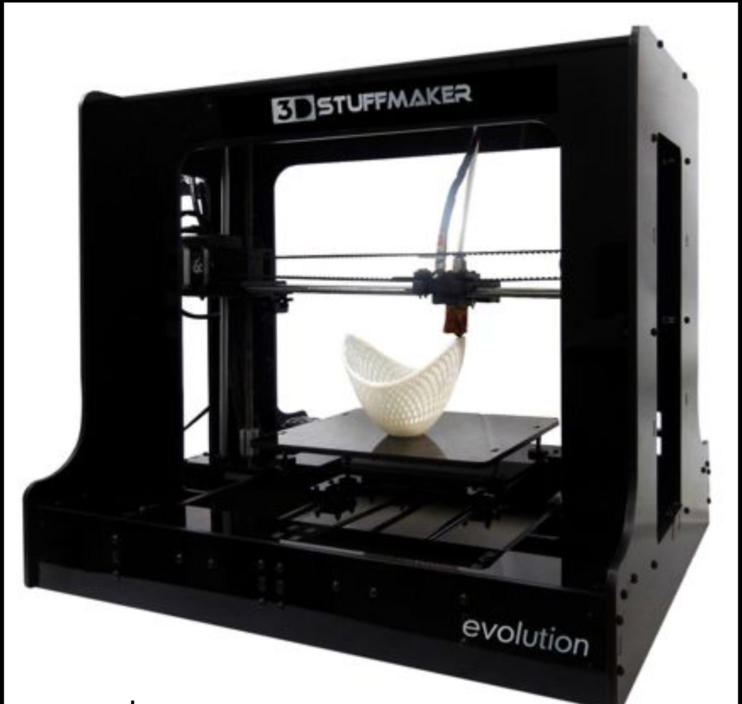
THE SOLUTION – *GOING A STEP FURTHER*

What if students could:

- SEE More
- FEEL More
- DO More

with the aid of Custom 3D-printed learning aids?

WHAT IS 3D PRINTING?



www.ebay.com

3D printers make manufacturing digital, personal and affordable

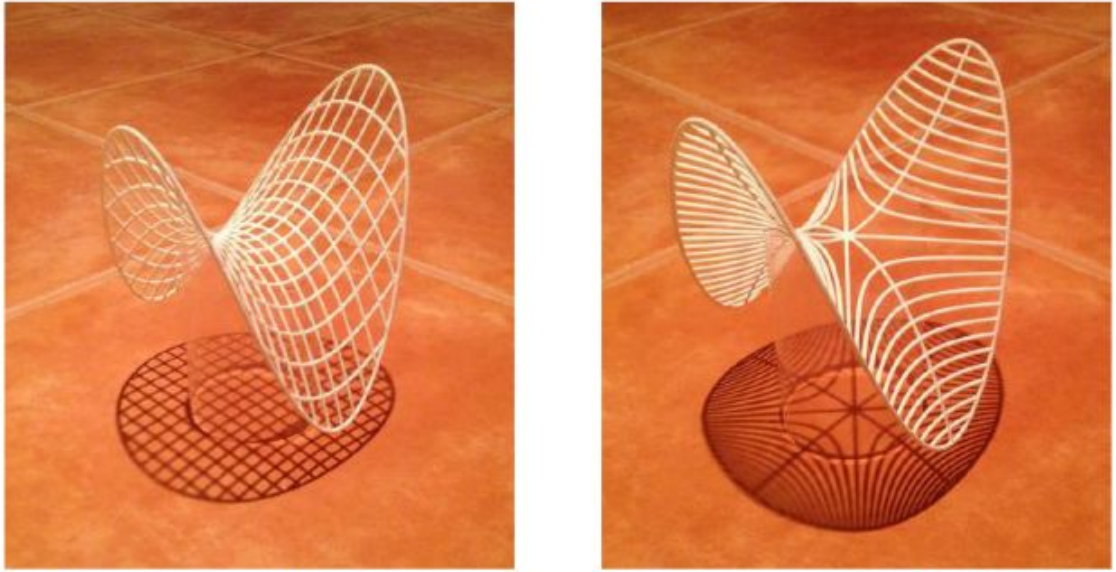
3D-PRINTED LEARNING AIDS

EXAMPLES OF CALCULUS SURFACES

Credit: Henry Segerman
youtube.com/watch?v=10KjAi5eA1Q

HOW TO PRINT CUSTOM 3D OBJECTS FOR TUTORIAL PROBLEMS: JUST 3 STEPS

MATHEMATICAL CONCEPT ⟶ COMPUTER MODEL ⟶ 3D PRINTED AID

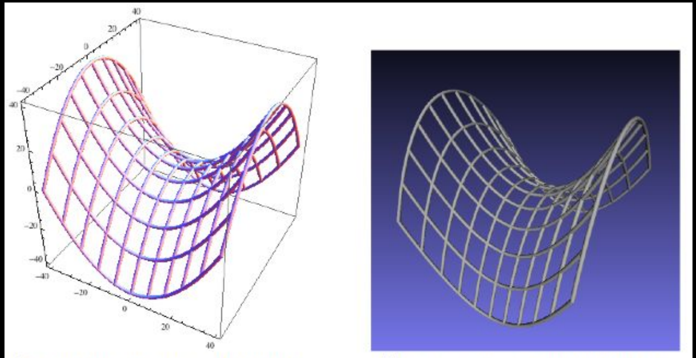


(a) A hyperbolic paraboloid, showing slices through the surface in the x and y directions.

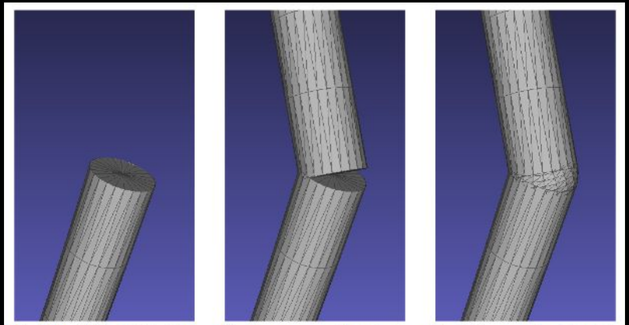
(b) A hyperbolic paraboloid, showing level curves.

HOW TO PRINT CUSTOM 3D OBJECTS FOR TUTORIAL PROBLEMS: JUST 3 STEPS

MATHEMATICAL CONCEPT → COMPUTER MODEL → 3D PRINTED AID



(a) Graphical output from Mathematica. (b) The STL file as viewed in MeshLab.



(a) The "Tube" PlotStyle directive produces closed polygonal meshes. (b) At a corner, these leave an ugly gap. (c) This can be fixed by adding a sphere at the corner.

12 lines of MATHEMATICA

Code

STL file for printing

```
f[u_, v_] := {u, v, u^2 - v^2};
scale = 40;
radius = 0.75;
numPoints = 24;
gridSteps = 10;
curvesU = Table[scale*f[u, i], {i, -1, 1, 2/gridSteps}];
curvesV = Table[scale*f[j, v], {j, -1, 1, 2/gridSteps}];
tubesU = ParametricPlot3D[curvesU, {u, -1, 1}, PlotStyle -> Tube[
    radius, PlotPoints -> numPoints], PlotRange -> All];
tubesV = ParametricPlot3D[curvesV, {v, -1, 1}, PlotStyle -> Tube[
    radius, PlotPoints -> numPoints], PlotRange -> All];
corners = Graphics3D[Table[Sphere[scale f[i, j], radius], {i, -1, 1,
    2}, {j, -1, 1, 2}], PlotPoints -> numPoints];
output = Show[tubesU, tubesV, corners]
Export["MathematicaParametricSurface.stl", output]
```

HOW TO PRINT CUSTOM 3D OBJECTS FOR TUTORIAL PROBLEMS: JUST 3 STEPS

MATHEMATICAL CONCEPT → COMPUTER MODEL → 3D PRINTED AID

Figure 9: The 3D printed object.

3D-PRINTED LEARNING AIDS

Scaling to suit classroom and tutorial style learning

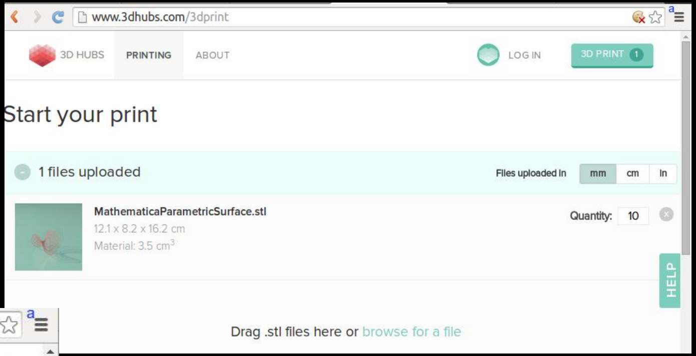
Credit: Henry Segerman

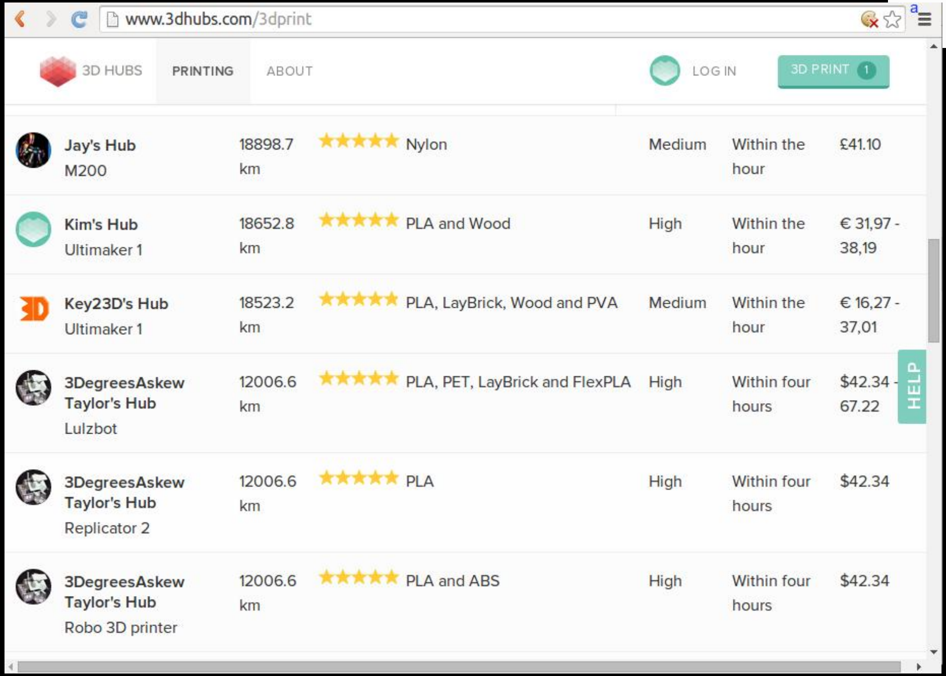
MORE MATH CONCEPTS THAT CAN BE AUGMENTED WITH LEARNING AIDS

- 2D & 3D Integration
- Volumes of Rotation
- Line & Contour Integrals
- Polar & Spherical Coordinates
- Transformation of random vectors
- Tangent Planes
- Linear Independence

FEASIBILITY

- Use online services





Average Tutorial:

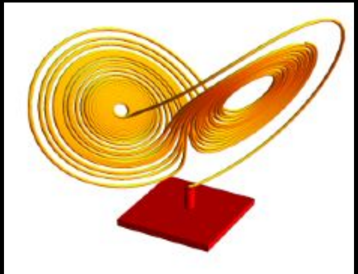
6 Problems

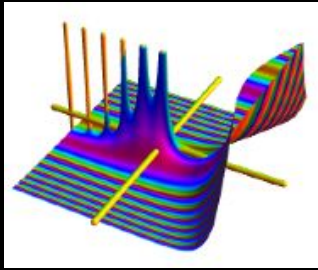
3 Parallel Sessions

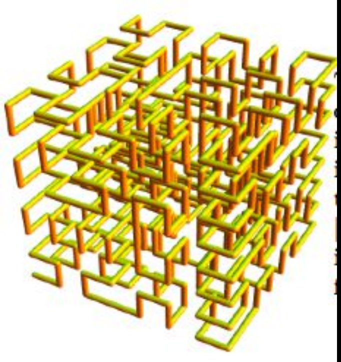
Will need 18 Objects

FEASIBILITY

- Use the printer outside Angstrom Library
  - Cost effective in the long run
  - More applications for
    - Honours courses



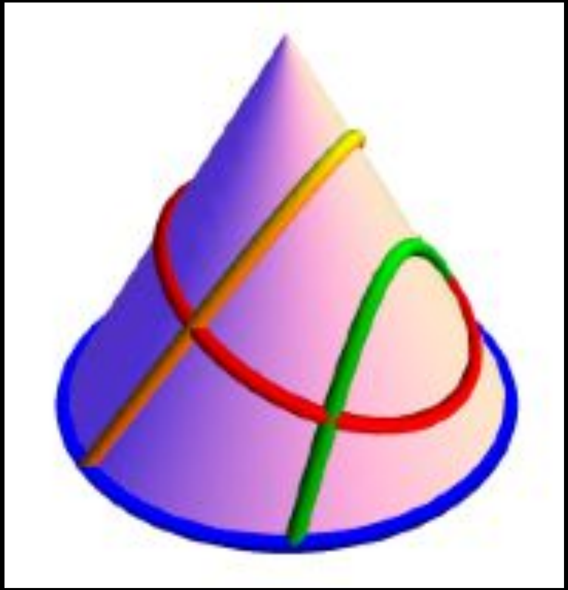




MEASURING EFFICACY

- Solicit Direct Feedback from
  - Students on their learning experience in the Course Survey
  - Tutors on Student Engagement
- Compare student performance in examinations with historical records

CONCLUSION



For conceptual clarity, there's nothing better than being able to see, feel and manipulate a mathematical object with one's bare hands!

REFERENCES

SEGERMAN H (2012) 3D Printing for Mathematical Visualisation.

KNILL O and SLAVKOVSKY E (2013) Illustrating Mathematics Using 3D Printers.

Thank you

End of Part 0

Part I: Kinaesthetic & Visual Cognitive Aids in Prob & Stats

How do we enable students to concretely understand Kolmogorov's axiomatic language of probability theory and ensuing statistical experiments?

- Now, the mathematical communication, even for the simplest experiments, is an order of magnitude more subtle
- We would like to use physical objects as well as the computer in complementary ways
- Let's dive into Galton's Quincunx and its extension known as the Septcunx
  - https://escholarship.org/uc/item/8b15m415
  - Erik Gustavsson will present his prototype of the Septcunx

Erik Gustavsson will present his prototype of the Septcunx

End of Part I

Part II: A Measurable Double Pendulum & Other Measurable Experiments

What about more sophisticated nonlinear experiments that are "chaotic", i.e., highly sensitive to initial conditions that can be modeled by nonlinear differential equations with noise?

Let's briefly deep dive here:

- A Mechatronically Measurable Double Pendulum

Other possibilities as "mathematical art"

- Pingala's Fountain at Chennai Mathematical Institute https://youtu.be/qHhSbJSTDLk
- Malkus Water Wheel governed by Lorentz Eqns. https://youtu.be/JxKAxojliKY
- Doubochinski's Pendulum, etc.

End of Part II

Part III: SageMath for Theorems & Live Data in Data Science

SageMath to illustrate instances of Probability Theorems and Statistical Models in the Introduction to Data Science Course that Benny Avelin and I are currently teaching.

UU Course 1MS041, Live Demo Now!

https://uppsala.instructure.com/courses/44135/pages/jupyter-notebooks?module_item_id=301399

Questions & Discussions