

# Holographic Vortex Dynamics of Pulsars

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16 May 2014

Outline

Motivation &  
Background

Physics of  
Superfluid  
Vortices

Gauge-Gravity  
Duality

Superconductivity  
3+1 Holographic  
Dual Model

Applications

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- 1 Motivation & Background
  - Physics of Superfluid Vortices
- 2 Gauge-Gravity Duality
  - Superconductivity
  - 3+1 Holographic Dual Model
- 3 Applications
  - Developments & Scope

# Pulsar Glitching

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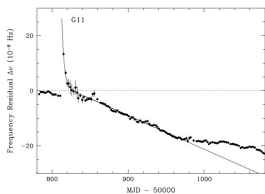
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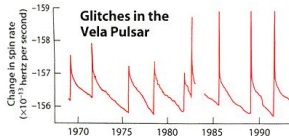
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- The sudden “speedups” in rotation period of a rotating Neutron Star (Pulsar); relaxing back in days to years, with no significant change in pulsed electromagnetic emission is referred to as Pulsar Glitching.



A: Glitch  $\frac{\Delta\Omega}{\Omega} \sim 9 \times 10^{-9}$  in the Crab

pulsar



B: Glitches in Vela Pulsar

Image Credits (A): Wong, T. et al. “Observations of a Series of Six Recent Glitches in the Crab Pulsar”. In: Astrophysical Journal 548, pp. 447

Image Credits (B): Courtesy Pulsar Astronomy by Andrew G. Lyne and Francis Graham-Smith

# Pulsar Glitching & Superfluidity

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- Since the pulse structure is not notably affected by glitch it must be phenomenon internal to the neutron star. Long time scales for response indicate well-oiled machinery – superfluidity! [*Metastable superfluid flow*, Packard 1972]
- A superfluid is one of the realizations of a Bose Einstein Condensate and has both zero viscosity and zero entropy. It involves the formation of paired neutron pairs forming a boson, implying the macroscopic fluid obtains bosonic properties, enabling the formation of a BEC described by a single coherent quantum wave function: first observed in helium II. <sup>3</sup>

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<sup>3</sup>Kapitza, P.L., 1938, Dk.Akad.Nauk SSSR, 18, 21

# Superfluidity & Vortices as potential candidates

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- The Landau criterion for superfluidity necessitates

$$\nabla \times \vec{v}_s = 0 \quad \forall \vec{v}_s \neq 0 \text{ where } \vec{v}_s$$

is the superfluid velocity.

- However, the macroscopic rotation in a superfluid that is experimentally observed is reasoned by regarding the “void regions” inside the integration contour of the superfluid to be multiply connected. This gives rise to the concept of a vortex core; a non-superfluid region with rotational flow within the surrounding the irrotational superfluid matter. The circulations with non-zero integral multiples of  $2\pi$  then give rise to

$$\kappa = \oint_{\mathcal{C}} \vec{v}_s \cdot d\vec{l} = \frac{2\pi\hbar}{m_{n,p}} \sim k \frac{h}{m} \Big|_{k \in \mathbb{Z}}$$

quantized vortex line as

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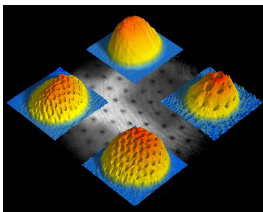
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- According to the theory related to neutron stars proposed by Ginzburg and Kirzhnits in 1964<sup>4</sup> - if rapidly rotating neutron stars exist then quantized vortices similar to those observed in helium II should exist in the neutron superfluid of these compact bodies.

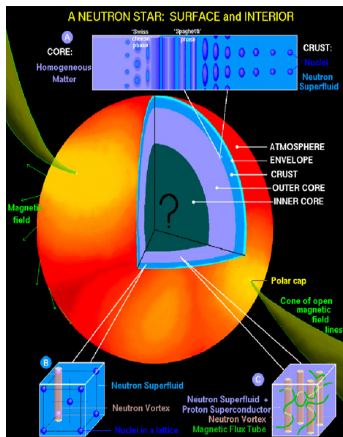


Arrays of vortices that form in spinning BECs resemble the insides of neutron stars

<sup>4</sup> Ginzburg, V.L., Kirzhnits, D.A., 1964, Zh. Eksp. Teor. Fiz., 47, 2006

Image Credits: Swiss cheese-like gas cloud holds clues to star-quakes in the Crab Pulsar  
Media Relations Office JPL, California, April 3, 2001 <http://www.jpl.nasa.gov>

# Vortex Pinning and Vortex Creep in Neutron Stars

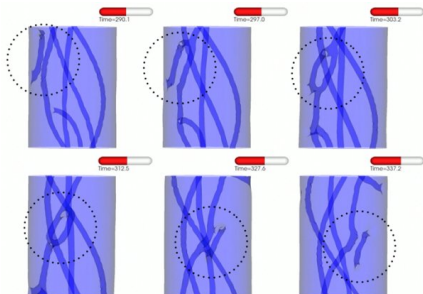


Neutron star model

The macroscopic superfluid velocity is determined by the spatial arrangement (fixing or pinning) of these vortex lines to the crust. At relatively low densities of the superfluid, we have vortex lines threaded between nuclei called *interstitial pinning* as opposed to the high densities of the crust where we have *direct nuclear pinning*.

# Vortex Pinning and Vortex Creep in Neutron Stars

- The concepts of Vortex pinning and Vortex creep in the inner crust of a Neutron Star are essential in many glitch mechanisms. Vortex creep is the process whereby pinned vortices unpin from one pinning site, migrate radially outwards, and re-pin to another suitable site.<sup>78</sup>



Fermionic vortex crossings and exchanges in an excited system of cold atoms in an elongated trap: akin to the pinning and de-pinning vortex mechanisms in neutron stars

Image Credits: Science, 10 June 2011: Vol. 332 no. 6035 pp. 1288-1291

<sup>7</sup> Teor. Fiz., 47, 2006 Anderson, P.W., Itoh, N., 1975, Nature, 256, 25

<sup>8</sup> Alpar, A., Anderson, P.W., Pines, D., Shaham, J., 1984, ApJ, 276, 325



# Vortex-Creep Model in Neutron Stars

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The general understanding of the vortex-creep model is -

- Vortex-pinning constrained the angular momentum of the superfluid leading to a lag between the angular rotation rates of the charge-less crustal superfluid and the charged normal-matter of the crust (driven by the electromagnetic external torque)
- The resulting lag in velocities determined the dynamics of the hydrodynamic lift (Magnus) force and the pinning force experienced by the inner- crust superfluid vortices leading to different pinning scenarios -
  - perfect pinning regime (frozen vorticity)
  - imperfect pinning with radially outwards gradual vortex-creep
  - sudden and catastrophic unpinning of a large number of vortices (around  $10^7$  to  $10^{14}$  out of about  $10^{18}$  in total) causing free angular momentum transfer of stored vorticity from the inner-crust superfluid to observable normal crust (magnetically strongly coupled outer-crust and charged plasma of the interior) - a characteristic giant glitch

# Superconductor Models

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- An exhaustive amount of theoretical models, including core-driven models as opposed to this two-component model have tried to explain the physics of these vortices; however, the entirety of physics of pulsar glitches (including their post-recovery physics) due to vortex pinning have not be thoroughly explored.
- Based on the theory of superconductors by Landau and Ginzburg<sup>9</sup>, describing superconductivity in terms of a second order phase transition whose order parameter is a complex scalar field  $\varphi$  (The density of superconducting electrons is  $n = |\varphi|^2$ ).

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<sup>9</sup>V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. 20 (1950),1064

# Physics of Superconductor Models

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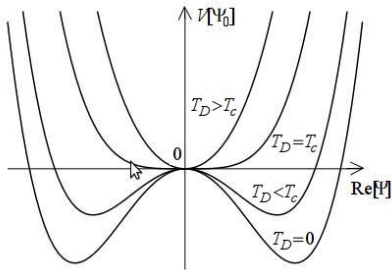
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- For  $T > T_c$  the minimum of the free energy is at  $\varphi = 0$ , while for  $T < T_c$  the minimum is at a nonzero value of  $\varphi$  - a fact similar to the Higgs mechanism in particle physics with the spontaneous breaking  $U(1)$  symmetry.



The schematic temperature dependence of the potential  $V[\psi_0]$  for a second-order phase transition

Image Credits: Roton energy gap and spontaneous symmetry breaking.

//arxiv.org/abs/0911.1599v1

# Need for Holographic descriptions

## Holographic Vortex Dynamics of Pulsars

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- Even with the case of BCS superconductors<sup>10</sup>, where interactions with phonons can cause pairs of electrons with opposite spin to bind and form a charged boson called a “Cooper pair”, below a critical temperature  $T_c$ , there is a second order phase transition and the Cooper pair, being bosons, condenses giving rise to superconductivity.
- However, there is evidence that electron pairs still form in relatively higher  $T_c$  materials, but the pairing mechanism is not well understood as it involves *strong coupling*.

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<sup>10</sup>J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Microscopic Theory of Superconductivity, Phys. Rev. 106, 162 - 164 (1957)

# Gauge-Gravity duality

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- A frontier based on exploring the ideas of a “duality” or “holography” between two hitherto unrelated energy regimes of physics can provide some insightful ideas to understand the underlying physics!
- The AdS/CFT correspondence has become a powerful tool to study strongly-coupled systems in different environments including in the case of certain condensed matter systems.

# Holographic Superconductor

## Holographic Vortex Dynamics of Pulsars

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### Physics of Superfluid Vortices

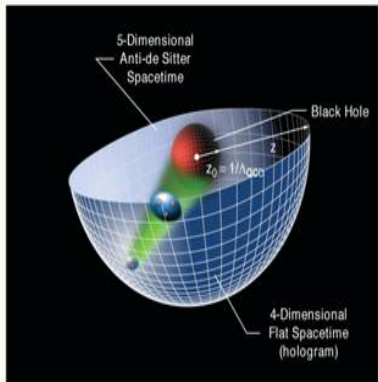
### Gauge-Gravity Duality

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### 3+1 Holographic Dual Model

### Applications

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GRC  
August 2, 2006

AdS/QCD  
3

Stan Brodsky, SLAC

AdS/CFT correspondence<sup>11</sup> is essentially a duality between two theories: a weakly coupled theory with a gravity defined in higher dimension (AdS Space with constant negative scalar curvature) and the other a strongly coupled gauge theory (CFT) without gravity, essentially a dimension lower on the conformal boundary.

<sup>11</sup> J. M. Maldacena, The large N limit of super conformal field theories and supergravity, Adv. Theor. Math. Phys. 2, 231 (1998)

Image Credits:

<http://www.slac.stanford.edu/th/Brodsky/BrodskyHome.html>

# Holographic Superconductors

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- A holographic description of a finite temperature CFT (superconductors) requires the temperature to be introduced by adding a charged AdS black hole solution and a condensate through a charged scalar field. The charge of the black hole also solves the no-hair dilemma of black holes<sup>13</sup>horizon
- A gravitational description of a superconductor/superfluids proposes models where a gauge  $U(1)$  symmetry is broken by a scalar field that turns on near the black-hole horizon. This corresponds, in the dual description, to a global  $U(1)$  broken by a scalar condensate. Therefore, strictly speaking, these models describe either a superfluid or a superconductor in the gauge-less limit.

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<sup>13</sup>S.S. Gubser, Breaking an Abelian gauge symmetry near a black hole

# Holographic Superfluid Model

- The simplistic model<sup>14</sup> consists of a charged scalar  $\Psi$  coupled to a  $U(1)$  gauge field  $A_\alpha$  in  $3 + 1$  dimensions ( $\alpha, \beta = 0, 1, 2, 3$ )

$$S = \int d^4x \sqrt{g} \left\{ R + \frac{6}{L^2} - \mathcal{L} \right\}$$

and an action given by –

$$\mathcal{L} = -\frac{1}{4} \mathcal{F}_{\alpha\beta}^2 - |D_\alpha \Psi|^2 - m^2 |\Psi|^2$$

- This is just general relativity with a negative cosmological constant (asymptotic AdS Radius,  $L$ )  $\Lambda = -\frac{3}{L^2}$  coupled to a Maxwell field and charged scalar with mass  $m$  and charge  $q$ .

$$\mathcal{F}_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$$

Also as standard procedure,

$$D_\alpha = \partial_\alpha - iqA_\alpha$$

<sup>14</sup> Introduction to Holographic Superconductors: Gary T. Horowitz



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- We like to work in probe limit where we can neglect the backreactions of the scalar field and gauge field on the background geometry.
- Starting from AdS-Schwarzschild black hole metric given as –

$$ds^2 = -ge^{-\chi} dt^2 + \frac{dr^2}{g} + r^2(dx^2 + dy^2)$$

and an ansatz of

$$A_\alpha = \phi(r)$$

and

$$\Psi = \psi(r)$$

within the probe limit,  $q \rightarrow \infty$   
and  $qA$ ,  $q\Psi$  are finite.

# Holographic Superfluid Model

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- On solving the differential equations for the gauge field and scalar field on the AdS-boundary, we acquire the corresponding fields external to the CFT.
- Reading off the AdS/CFT dictionary we can then establish that these fields are coupled to the CFT operators through interaction terms. Integrating over the CFT fields, one can obtain the free energy from which the vacuum expectation values (VEV) of the CFT operators can be extracted.
- The dual CFT theory, if it exists, is supposed to be strongly coupled and the limit  $g \rightarrow 0$  in the AdS theory corresponds to be working at the planar level in the CFT.

# Comments and Merits

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- The bulk gravitational theory was just the simplest model that could describe a dual superconductor and it works rather well : it predicts that a charged condensate forms at low temperature (in a second order phase transition), that the DC conductivity is infinite, and that there is a gap in the optical conductivity at low frequency - all basic properties of superconductors!
- Since the bulk dual of a superconductor just involves gravity interacting with a Maxwell field and a charged scalar, there is a superficial similarity to a Landau-Ginzburg description. However, it is important to keep in that the low temperature instability in the GL theory must be put in by hand, whereas it arises naturally in the gravitational description.

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- Also, the GL model is only valid near the transition temperature, since it involves a power series in the order parameter  $\varphi$ . To go beyond  $T \sim T_c$  one would need to specify an entire potential  $V(\varphi)$ , which wasn't a requirement here; expect when one embeds into string theory.

# Continuing Work

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- Using the established gauge/gravity duality one can begin to test various properties associated with vortices including decay of magnetic field which could then be corroborated with the established theory of vortex dynamics lead pulsar glitches.
- Moreover, using the simplified holographic model itself one can try and talk about the physics relating pinning of vortices to the crust.
- One can also correlate the conventional properties of a simplistic 2+1 gauge theory to the dual vortex theory and try and understand what a vortex-like solution would correspond in the bulk with respect to cosmic strings at the domain wall intersection on the boundary.

# Thank you for your time & your patience!

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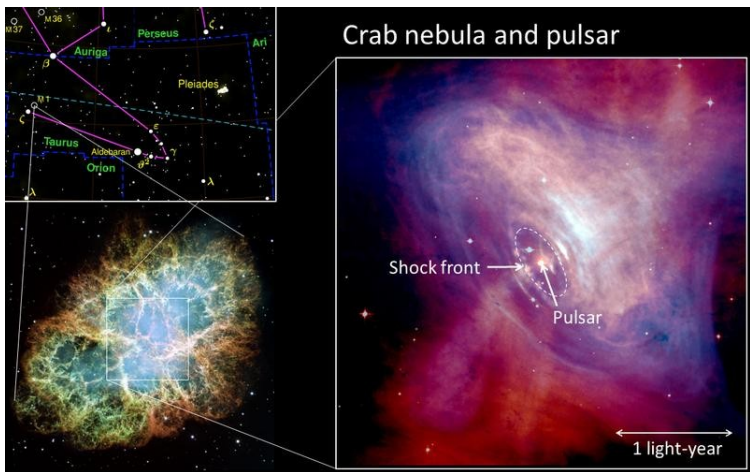


Image Credits: MPIK, source: NASA The Crab nebula (M1) in the Taurus constellation, taken by the Hubble space telescope (lower left). The details to the right are showing a composite of visible light (red) and X-rays (blue) with the pulsar as central star. At the shock front in 0.3 light-years distance from the pulsar, the ultra-relativistic wind of electrons and positrons collides with the surrounding nebula.