Sangaku - Japanese Temple Mathematics

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What are Sangaku

- Sangaku are wooden tablets containing mathematics
- Traditionally hung in eaves of Shinto shrines and Buddhist temples
- Practice of dedicating sangaku began during the early Edo period (1600-1868)
- This was during the period of national seclusion
- Earliest known tablet dedicated in 1683
- Many significant sangaku were dedicated during the Meiji period (1868-1912)
- Sangaku using modern mathematics are still produced
Features of Sangaku

• Written in the **Kanbun** language rather than ordinary Japanese
• Kanbun was a rather prestigious academic language like Latin
• Many Confucian texts and literary works used this language
• Often contain geometrical problems
• Problems are generally abstract, but not exclusively (e.g. finding the height of sacred mountains)
• Commonly give a problem and a proof
• Sometimes challenge the observer to figure out the proof
• Sometimes the creator can’t solve the problem
• Created by everyone – professional mathematicians, samurai, farmers, women, and children
• Contain colourful accompanying pictures
• Sometimes contain non mathematical paintings
Katayamahiko Shrine, Okayama Prefecture
August 1873
Toenji Shrine, Tokyo Prefecture, 1869
Sangaku from Yamagata Prefecture, 1823
京都府，1683年

奉納等法

昭和五十二年五月二十四日

山本宗信

天和三年文年

伏見役山御書室に

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A circle of radius \( r \) inscribes three circles of radius \( t \), the centers of which form an equilateral triangle of side \( 2t \).

Find \( t \) in terms of \( r \).

If \( r = 10 \), then \( t = 4.64 \) \hspace{1cm} (SMJTG p. 96)
Solution

• We know the distance between \( r \) and any \( t \) is \( r - t \)

• We know the centres of \( t \) form the corners of an equilateral triangle, so \( \cos 30^\circ = t / (r - t) \)

• We can describe \( t \) in terms of \( r \) as so

\[
t = \frac{\sqrt{3}r}{2 + \sqrt{3}} = (2\sqrt{3} - 3)r = 0.464r
\]

• Most sangaku deal with problems of this form
An equilateral triangle with a side $a$ is inscribed in a square also of side $a$, along with four circles, including two of radius $t$ and one of radius $r$.

Find $t$ in terms of $r$. (SMJTG p. 114)
• Need to recognize the triangle is equilateral
• \[ \tan 30^\circ = \frac{2r}{a} = \frac{1}{\sqrt{3}} \]
• Drawing a line from say the center of the right circle \( t \) to the lower right-hand corner shows that \[ \tan 15^\circ = \frac{t}{a-t} \]
• A good half-angle formulae for tangent in this case is \[ \tan \left( \frac{\theta}{2} \right) = 1 - \cos \theta \] giving \[ \tan 15^\circ = 2 - \sqrt{3} \]
• Eliminating \( a \) from the two expressions gives \[ t = (\sqrt{3} - 1)r \]
• Trick is looking for what angles we have and working from there – that is the traditional way
• Intuition and deduction are important
Sawa Masayoshi’s problem

- An ellipse is inscribed in a right triangle with its major axis parallel to the hypotenuse. Within the ellipse are inscribed two circles of radius $r$. A third circle of radius $r$ touches the ellipse and the two shorter sides of the triangle, $a$ and $b$.

- Find $r$ in terms of $a$ and $b$ (SMJTG p. 259)
Why create Sangaku?

• Sangaku are a result of the idai tradition
• Idai are difficult problems without answers
• First appearance was in the 1641 edition of the Jinkouki of Yoshida Mitsuyoshi
• The Jinkouki taught mathematics to the general public. It focused on commercial problems and was in a problem-answer format
• Jinkouki was extremely popular, estimated to have sold over 1 million copies. Many illicit copies were produced.
• To curb plagiarism, author added a section of difficult problems and challenged the copiers to solve them
Competitive Mathematics

• Soon after the inclusion of idai in the Jinkouki, they began to appear in other mathematical texts

• Idai became a way for those apt at mathematics to test and show off their abilities in the absence of a unified discipline of mathematics

• The few schools of mathematics that existed were secretive and competitive

• Sangaku most likely developed as a local way to present idai
Why Shrines and Temples?

- Sangaku are related to *ema*
- Ema are wooden tablets which initially would display pictures of horses (e = picture, ma = horse)
- Horses used to be sacrificed to the Gods, but it was expensive so people started to dedicate pictures instead
- Many were created that also displayed painted scenes of mythological or historical significance
- They were also hung in eaves and in shrines and temples
- Modern ema are smaller – one puts their wish/hope on the back, hangs it up, and they are burned
Old ema hung in a Shinto shrine
• Because they take the form of ema, they likely had some religious function
• We see a connection to religion in the preface to the following sangaku from 1815 for example

“The teacher Takeda has been studying mathematics since he was young. In this shrine, his disciples ask God for progress in their mathematical ability and dedicate a sangaku” (SMJTG p.243)

• People believed the Gods might give them good mathematical luck or abilities by dedicating work to them
• Shrines and temples were public places of community as well
• Hanging a sangaku in a shrine or temple ensured maximum publicity
Why Geometry?

• It may have seemed more appropriate to present mathematics which was highly visual like this so it fitted in with other artistic dedications
• May have created a sense of wonder in non mathematicians and functioned as objects of contemplation and awe
• May have also pleased and attracted the Kami
• Circles are associated with the sun goddess - heavy use of circular problems could be paying homage to her
• Geometry is generally a popular area of study across cultures historically, so perhaps there is no other reason than that people are naturally drawn to it
Interest for Historians

• The tradition developed in recent times so we can easily pinpoint its beginning and evolution

• It is an example of mathematics developing in relative isolation

• It is an example of an interesting unique way of expressing mathematics – combining of mathematics, art, and religion

• They are reflective of Japanese culture of the time and attitudes towards mathematics

• Gives us a sense of their abilities and thoughts
Sangaku Revival

• In the last twenty years, there is a small revival in sangaku production
• Teachers are starting to use sangaku to introduce geometry to students
• New style of sangaku keep the traditional design but often use modern mathematical notation and everyday Japanese instead of kanbun
• Here are some examples
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 Sangaku created by middle schooler in 2006
Further reading/references

• *Sacred Mathematics: Japanese Temple Geometry* by Tony Rothman & Hidetoshi Fukagawa

• ‘Japanese Temple Geometry’ in *Scientific American* (May 1998)

• Hiroshi Kotera’s website [www.wasan.jp](http://www.wasan.jp) (Japanese version is recommended – has heaps of great images)

• There is also a facebook page run by Hiroshi Kotera – search for 和算なう