Primer on glacier flows

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1. Why is it important to understand the physics of glaciers?

- Glaciers are history books of nature
  And contain many information about palaeontologic times
  - about climate changes
  - ocean sediments (ice shelfs in antarctica)
  - pollen
- Glaciers contribute to the raise/drop of the seawater level
- Glaciers are the biggest reservoir of fresh water on earth (74%)
  groundwater 20%, lakes, rivers 0.36%

Understanding of glaciers is also important for example to prevent nature catastrophes (glacier surges) or to use the water of glaciers in power plants or for water supply for agriculture.

2. The basics of glacier and ice sheet motion

What contributes to the glacier dynamics?

- Precipitation and the melting
- local climate
- Temperature distribution
  - consistency of the ice
  - does the glacier slide or is it frozen to the rocks?
- Geometry of the bedrock (flat, steep, wide (Antarctica))
- ….

Consider the easiest case of an isothermal glacier. The motion of every continuum (be it a liquid or a solid) is described by the momentum balance and the mass balance:

Mass balance

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\vec{u}\rho) = 0$$

Momentum balance

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right) = \rho\vec{g} + \nabla\cdot\mathbf{T}$$

where

$\vec{x}, t$	space and time
$\rho(\vec{x}, t)$	density

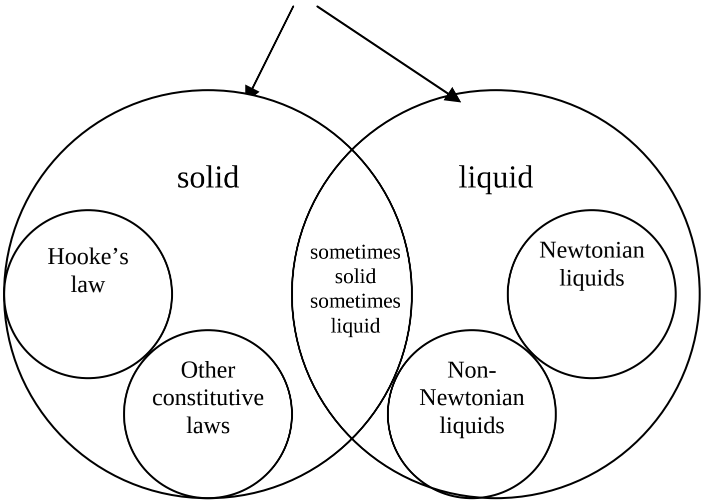
$\vec{u}(\vec{x},t)$	velocity vector
$\vec{g}$	gravity vector
T	stress tensor/matrix (usually symmetric, i.e. 6 degrees of freedom)

Assume that density is constant $\rho = const$, then we have 4 equations and 9 unknowns. This system is underdetermined and we need more equations.

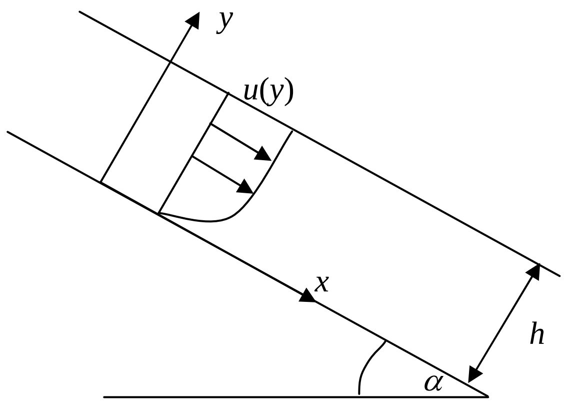
Material science:

constitutive relations between the stress tensor and the strain rate

$$\mathbf{T} \sim \dot{\varepsilon} \sim f(\vec{u})$$



Observations: glacier "flow" at a very long timescale.



Constitutive equation

$$\dot{\varepsilon}_{ij} = A\tau^{n-1}\tau_{ij} \text{ (Glen's law)}$$

with $\tau^2 = \tau_{13}{}^2 + \tau_{12}{}^2 + \tau_{23}{}^2 + \frac{1}{2}\left(\tau_{11}{}^2 + \tau_{22}{}^2 + \tau_{33}{}^2\right)$, the second invariant of the stress tensor.

Reduced to one dimension (only one component of the shear rate is non-zero) the momentum balance

$$0 = \rho g \sin\alpha + \frac{\partial \tau_{12}}{\partial y}$$

Which can be integrated once to

$$C_1 - \rho g \sin\alpha\, y = \tau_{12}$$

Glen's law in 1 dimension reads

$$\frac{\partial u}{\partial y} = 2A\tau_{12}{}^n$$

Integration constant is determined by $u(y=0)=0$: no-slip at the bedrock (i.e. glacier is frozen to the rock)

This gives

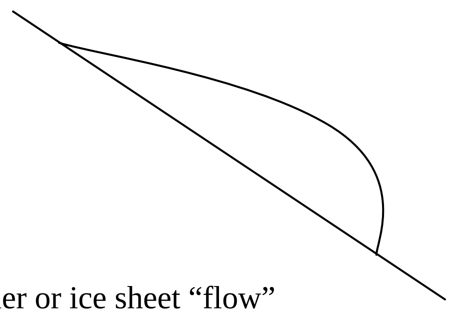
$$u = \frac{2A(\rho g \sin\alpha)^n}{n+1}\left(h^{n+1} - (h-y)^{n+1}\right)$$

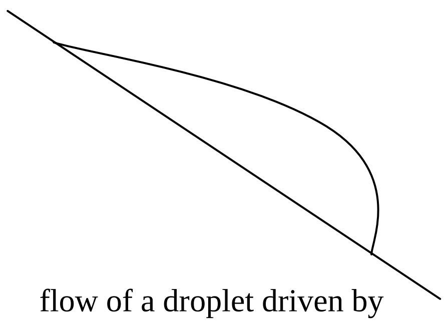
A parabola of degree $n+1$

In reality:

- Glacier thickness h is not constant
- Glen’s flow parameter depends on the coordinates and temperature
- Precipitation/snowfall/melting is not negligible

3. Analogy to film flow





Glacier or ice sheet “flow”

flow of a droplet driven by gravity

Typical dimensions for glaciers
length about 20 km
height about 100 m
ice sheets: length 3000km, height 3km

Typical dimensions for droplets
length: mm to micrometers
height: micrometers to nanometers

Length scales suggest a shallow ice approximation

Length scales suggest a shallow water (thin film) approximation