• Rheology :

Science dealing with the flow and deformation of materials experiencing some mechanical force.

(Eugene) and Markus Reiner.
Professors E. C. Bingham coined the term in the late 1920's.

It comes from the Greek

$\rho\varepsilon o\varsigma$ ← sigma ( logia ) study

which means stream.

and took the motto of the subject the statement by the Greek philosopher Heraclitus

$\pi\alpha\nu\tau\alpha\ \rho\varepsilon\iota$

Everything Flows. (Society of Rheology.

• What does this mean ?

Judges 5:5 in the old Testament (Bible).

The prophetess Deborah stated that

The mountains flowed before the Lord.

• Two interesting facts.

1) The mountains flowed !!

(water flows but mountains ?).

• Rock slides, mud flows and all the
(mass wasting processes (slope movement))

geomorphic process by which soil, regolith and rock move downslope under the force of gravity.

Examples of the ways in which a mountain may flo

Although some geological flow processes may occur rapidly an violently (rock and snow avalanches)

a great many others are taking place almost imperceptibl

$\boxed{\text{before the Lord}}$

• All is a matter of time scales.

human timescale may be insufficient to see mountain flowing.

Example

Himalayas: as a result of the collision between the Indian sub-continent with the rest of Asia some 100 million years ago. these mountains are continuing to rise. (1 mm per year).

In a manner analogous to the piling up of butter (Him) in front of the spreading knife (sub-cont In

The fundamental difference between the two flow process of mountains and butter lies in the viscosities and timescales.

Conclusion $\Rightarrow$ distinction between solids and fluids now seems somewhat more blurred than we may have previously thought.

A discussion of material behaviour might be more appropiate.

In order to understand the behaviour of materials as fluid-like or solid-like it is helpful to discuss what relaxation-time and retardation-time of a material mean.

We begin by looking at the two extremes.

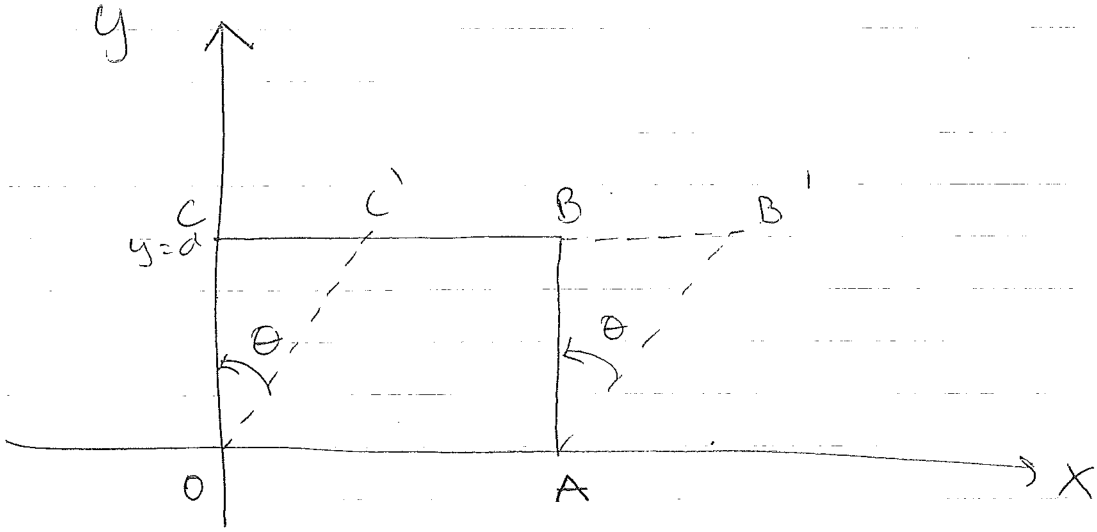
- Hookean elastic solid
- Newtonian fluid.

Consider a one-dimensional simple shearing motion in a

Hookean linear elastic solid which occupies at some time $t = T < 0$

$$\Omega = \{ (x, y, z) : -\infty < x < \infty,\ 0 \leq y \leq d,\ -\infty < z < \infty$$

in its equilibrium state.



No. OF QUESTION

Suppose that

OABC in any plane $z$ = constant undergoes a small deformation so that at time $t = t' > 0$ it is a parallelogram $OAB'C'$ with angle $\widehat{C'OC}$ denoted by $\theta(t')$ (idependent of $x, y, z$).

Consider a material particle having Eulerian coordinates

$$(x(t'), y(t'), z(t')) = (X(t'), y(T), z(T)).$$

Then we define the deformation gradient

$$\gamma_{xy}(T, t') = \frac{X(t') - X(T)}{y'(t')} = \tan(\theta) \simeq \theta(t')$$

for small $\theta$.

Thus $\theta$ is called the strain and Hooke's law states

rigidity moddlus of the material.

$$\underset{\text{shear-stress}}{\sigma_{xy}} = G\,\theta(t)$$

measure of how much a given displacement differs locally from a rigid body displacement

Suppose now that a Newtonian fluid occupies the region $\Omega$ between the same two planes $y=0$ and $y=d$ and that the material undergoes the same shearing deformation as before. Then, by Newton's hypothesis.

$$\sigma_{xy}(t) = \eta_0\, \dot{\gamma}_{xy}(t)$$

where

$\dot{\gamma}_{xy}(t)$ is the rate of strain at time $t$ defined

$$\dot{\gamma}_{xy}(t) = \frac{d}{dt}\gamma_{xy}(T, t) = \dot{\theta}(t)$$

No. OF QUESTION ~~We can think~~

In simple terms the relaxation time of a material may be defined as some appropriate measure of the time required for shear stress $\tau_{xy}$ in a simple shear flow to return to zero under constant-strain conditions.

- Hookean elastic linear solid will never relax to zero
  $\Rightarrow$ relaxation time is infinite.

- Newtonian fluid relaxation of the stress is immediate and the relaxation time is zero.

In reality this is a mathematical ~~ideali~~ idealization of Hookean elastic solids and Newtonian liquids.

In practice, stress relaxation after the imposition of constant-strain conditions ~~oc~~ takes place over some finite non-zero time interval.

- This is the defining characteristic of so-called viscoelastic materials.

To illustrate this behavior consider the following relationship between shear stress and rate of strain, (Maxwell model)

$$\lambda \frac{d}{dt}\tau_{xy} + \cancel{1}\,\tau_{xy} = \eta_0 \dot{\gamma}_{xy}(t)$$

dominant balance.

$\lambda >> 1$ Hookes law

$\lambda << 1$ Newtonian fluid.

NO. OF QUESTION

- The relaxation time of water is $10^{-12}$ s
- Low density polyethylene $10^{9}$ s
- Glass > 28hrs

The fact that a given material has a relaxation time associated with it does not mean that it will always behave as we may expect. What really matters in characterizing material behaviour in experiments is the ratio of a characteristic relaxation time of the material to a characteristic time of observation of the flow process.

this ratio is called Deborah number

$$De = \frac{\lambda_1}{T_0}$$

~~De~~

$\lambda_1$ non zero → material → solid short timescales.

flow if $T_0$ sufficiently large.

"Bouncing putty".

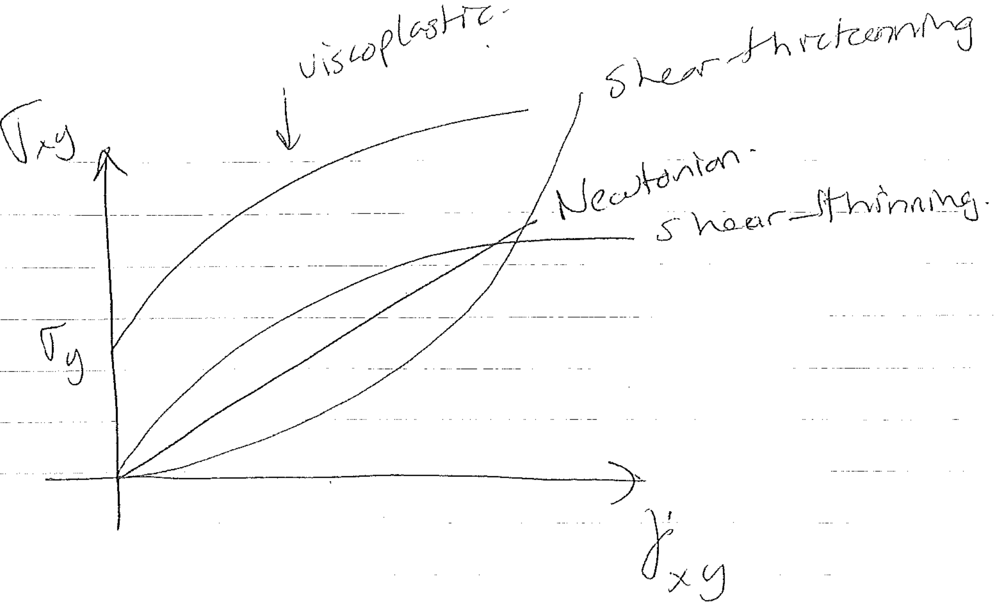
Under shear flow even if $De \ll 1$ the material
For ~~some materials~~
can exhibit Non-Newtonian behaviour.

The relationship between shear-stress and shear-rate is non linear:

Instead of $\sigma_{xt}(t) = \eta_0 \dot{\gamma}_{xy}(t)$

$$\Rightarrow \sigma_{xt}(t) = \eta(\dot{\gamma})\,\dot{\gamma}_{xy}(t)$$

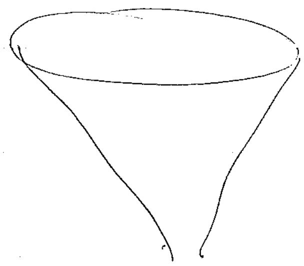
$$\dot{\gamma} = |\dot{\gamma}_{xy}|$$




(Pitch Drop experiment) 2005

Ig Nobel Prize in Physics.

Experiment began in 1927
One drop falls more less every 9 years, 8 drops so far.



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A solid is defined as a material that will not continuously change its shape when subjected to a given stress; for a given stress there will be a fixed final deformation which may or may not be reached instantaneously on application of the stress.

A liquid is defined as a material that will continuously change its shape when subjected to a given stress, irrespective of how small the stress.

No. OF QUESTION

## Constitutive eqns:

Theory.

Theorem (Renardy 1985)
Suppose that

1. $\Omega \subset \mathbb{R}^3$ is a bounded domain with boundary $\partial\Omega$ of class $C^3$

2. $b \in H^2(\Omega)$, $u_0 \in H^{5/2}(\partial\Omega)$ and the norms of $b$ and $u_0$ are sufficiently small in these spaces.

3. $u_0$ is parallel to $\partial\Omega$

Then $\exists$ a solution ~~ate~~ of the equations governing the flow of a UCM fluid.
The solution satisfies

$$u \in H^3(\Omega), \quad p, \tau \in H^2(\Omega)$$

and it is the only solution for which the norms in these spaces are small apart ~~for~~ from an arbitrary constant in the pressure solution.

Iterative approach in which one alternates between the solution of an elliptic problem of generalized Stokes type and a hyperbolic system that has the streamlines as characteristics.