Lecture 1 : Monday July 13, 2009.

Chit-chat

- Let us make sense of Google's Chief Economist's message.

(*) An experimenter ✡ observes ✡ articulates, processes and decides/act

- articulates → Expressing oneself Readily, clearly and effectively (Maths) "tell a story"
- processes → Computational or Algorithmic (Comp. Sci.)
- decides/act → "real world"

Statistician's Job Description

DATA ✡ Observe → Language ✡ articulate with maths → articulate → Tools ✡ Program or process → make decision / take action → ✡ Act Decisions or Actions

Fields Requiring Statistical Decisions or Actions

- Sciences (Biology, chemistry, Physics...)
- Engineering (Civil, Mech, Electrical,
- Management (Commerce, MBA, O.R, ...
- ⋮
- Government (Official stats, ...)
- Military. (Supply chains, O.R., ...)
- Information Technology (Learning, ...)

Egs. of Decisions/Actions:

- Hypothesis Testing
- Parameter Estimation
- Model Selection
- Operations Research
- Statistical Learning
- Filtering
- Prediction
- ⋮

Qn:

How can we SAY WHAT YOU MEAN and MEAN WHAT YOU SAY ?

Ans: — clearly this depends on who you are talking to...

Language of Mathematics is a good candidate to SWYM&MWYS

— unlike other human languages it evolves in an internally consistent way

— Analytical Poetry.

Wednesday July 15, 2009

Sets, Maps, (Numbers, Arithmetic)

{it is assumed that you know $+, \times, -, \div, \wedge, \ldots$ on integers, rationals & reals

A **set** is a collection of distinct elements.
We write a set by enclosing its elements by curly braces

Ex. The collection of • and o is $\{o, \bullet\}$

We can name this set A and write $A = \{o, \bullet\}$

Ex Qn: is $A' = \{o, \bullet, \bullet\}$ a set?

$G = \{\alpha, \beta, \gamma, \delta, \varepsilon\}$ is the set of first 5 Greek alphabets

The set which does not contain any element is the empty set.

$$\emptyset = \{\ \}$$

We say an element belongs to or does not belong to a set and write $\in$ or $\notin$

Ex.

$$\bullet \in \{o, \bullet\} \quad \text{but} \quad ✡ \notin \{o, \bullet\}.$$

We say a set C is a subset of a set D and write

$$C \subseteq D$$

SAGE $\subseteq \Leftrightarrow \leq$

if every element of C is also an element of D

Ex:

$$\{\bullet\} \subseteq \{o, \bullet\} \quad \text{and} \quad \{✡\} \not\subseteq \{o, \bullet\}$$

Set Operations:

We can add distinct new elements to an existing set by union operation.

ex

$$\{o, \bullet\} \cup \{✡\} = \{o, \bullet, ✡\}$$

Qn: $\{o, \bullet\} \cup \{\bullet\} \overset{?}{=}$

Union

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

read as "the set of all x such that x belongs to A or x belongs to B.

SAGE $\cup \Leftrightarrow |$

Intersection:

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

SAGE $\cap \Leftrightarrow \&$

Set Difference

$$A \setminus B = \{x : x \in A \text{ and } x \notin B\}$$

SAGE $\setminus \Leftrightarrow -$

Disjoint Two sets A and B are said to be disjoint if $A \cap B = \emptyset$

Complement

Given a universal set U,

$$A^c = U \setminus A = \{x : x \in U \text{ and } x \notin A\}$$

Ex

Fruits = {orange, banana, apple}

Colours = {red, green, blue, orange}

what is:

Fruits $\cap$ Colours ? Is it not $\emptyset$? !
[strings]

" $\cup$ "

" $\setminus$ "

Colours $\setminus$ Fruits

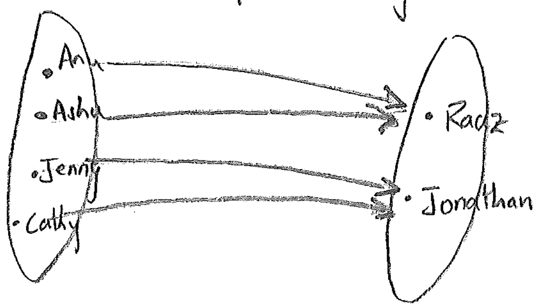
$\text{Colours}^c = ?$

A Map or Function is a specific kind of <u>relation</u> between two sets, traditionally called Domain & Range.

A <u>Map</u> or <u>function</u> associates each element in the domain with exactly one element in the range. So, two distinct elements in the domain can be associated with the same element in the range, and not every element in the range needs to be mapped.

<u>Ex</u>: findFathersMap : Daughters → Fathers

is equivalent to:
{(Anu, Raaz),
(Ashu, Raaz),
(Jenny, Jonathan),
(Cathy, Jonathan)}



Daughters (Domain)   Fathers (Range)

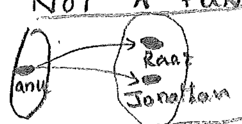
findFathersMap (Anu) = Raaz   "image of Anu"

⋮

" (Cathy) $\overset{?}{=}$

NOT A FUNCTION

Ex:



<u>Note</u>: A function $f : \mathbb{X} \to \mathbb{Y}$ is equivalent to the set, $\{(x, f(x)) : x \in \mathbb{X}\}$ ordered pair

The <u>pre-image</u> or <u>inverse image</u> of a function

$f : \mathbb{X} \to \mathbb{Y}$ is $f^{[-1]} : \mathbb{Y} \to \sigma(\mathbb{X})$ "a set of subsets of $\mathbb{X}$"   "sigma"

where, $f^{[-1]}(y) = \{x \in \mathbb{X} : f(x) = y\} \subseteq \sigma(\mathbb{X})$

"pre-image" or "inverse image" of y.

<u>Ex</u>

$\text{findFathersMap}^{[-1]}$ = findChildrensMap : Fathers → Daughters

"image of Anu and Ashu"

findChildrensMap (Raaz) = {Anu, Ashu}   "inverse image of raaz"

" (Jonathan) $\overset{?}{=}$

<u>Ex</u> $f(x) = x^2$

<u>Version 1</u> $f(x) = x^2 : \{-2, -1, 0, 1, 2\} \to \{0, 1, 2, 3, 4\}$

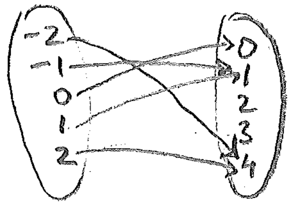
<u>finite</u> domain (only 5 elements)

<u>version 2</u> $f(x) = x^2 : \{\cdots, -2, -1, 0, 1, 2, \cdots\} \to \{0, 1, 2, \ldots\}$ more and more

infinite domain

Ex (Version 1 contd...).

$f(x) = x^2 : \{-2, -1, 0, 1, 2\} \to \{0, 1, 2, 3, 4\}$



this function with domain $\{-2, -1, 0, 1, 2\}$ and range $\{0, 1, 2, 3, 4\}$ is represented by the set of ordered pairs:

$\{(-2, 4), (-1, 1), (0, 1), (1, 1), (2, 4)\}$

Note. 2 and 3 in range have no pre-image in domain. But, this is o.k. since every element in the range need not be mapped.

Punch Line: We can make a computer-represent this function as finitely many ordered pairs.

(Version 2 contd...)

$f(x) = x^2 : \{\ldots, -2, -1, 0, 1, 2, \ldots\} \to \{0, 1, 2, 3, \ldots\}$

$\Updownarrow$

$\{\ldots, (-2, 4), (0, 0), (1, 1), (2, 4), \ldots\}$ is the corresponding set of ordered pairs. But it is impossible to write them all down. since there are infinitely many elements in the domain, so we cannot use a

Version 3

$f(x) = x^2 : \mathbb{R} \longrightarrow \mathbb{R}$. ← real line

Now, there are even more elements in the domain.

Punch Line Let $f : \mathbb{X} \to \mathbb{Y}$ be some function.

A computer with finite memory (all computers today) cannot represent a function with an infinitely large domain via the set of ordered pairs $\{(x, f(x)) : x \in \mathbb{X}\}$. So, in such cases, we implement or computer-represent the function as a procedure that will take as input or argument any $x \in \mathbb{X}$ and output or return its image $f(x)$.